Problem Solving

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1.0 Introduction

Problem solving is the process of applying previously acquired knowledge to obtain a satisfactory solution to new and unfamiliar problems. According to the National Council of Teachers of Mathematics (NCTM), "Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity" [1]. For engineering students, problem solving is so important to their success in engineering that in the ABET Engineering Criteria 2000, schools are required to demonstrate that their graduates have an ability to identify, formulate, and solve engineering problems [2].

Yet in a recently released international study examining reading, science and general problem solving skills of students [3], U.S. students "scored close to the bottom" in problem-solving skills. In another study of problem solving in engineering classes, Woods [4] found that during a four-year degree program, engineering students observe professors work over 1000 example problems, and the students themselves solve more than 3000 problems. However, when they graduate, students *show negligible improvement* in problem solving skills – meaning that if they were given a related but different problem situation, they were not able to solve it.

Even though problem solving has been identified as important, the way it is usually taught is to present students with a strategy for solving problems, such as the one in *College Algebra* by Mark Dugopolski [5]:

1. Read the problem as many times as necessary to get an understanding of the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable, write down what it represents, and if possible, represent any other unknown quantities in terms of that variable.
4. Write an equation that models the situation. You may be able to use a known formula, or you may have to write an equation that models only that particular problem.
5. Solve the equation.
6. Check your answer by using it to solve the original problem (not just the equation).
7. Answer the question posed in the original problem.

From this problem solving strategy, instructors then expect students to be able to solve all problems they are given during the quarter. The reason this method of teaching problem solving does not work is that knowing a problem solving strategy (which is what is listed above) is only one skill that successful problem solvers employ.

In a study of problem solving, Woods [6] has found that successful problem solvers have some or all of the following traits:

1. An awareness that a problem exists.

2. Prerequisite skills
    a. Basic knowledge pertaining to the problem area
    b. The learning skills necessary to obtain other information required for the solution
    c. Motivation to want to solve the problem
    d. Memorized experience factors that provide "feelings" about what assumptions might be made and how reasonable an answer is
    e. Ability to communicate the result
    f. Group skills, if a team approach is used
3. An overall, organized strategy
4. Alternatives for specific steps in the strategy (contradiction, reasoning by analogy, working backwards, solving a simpler problem first, etc.)
5. Knowledge of heuristics or "rules of thumb" that offer suggestions about what to do next.
6. Ability to create, to generalize and to simplify.

The organization of this paper is that in Section 2 the basic knowledge that students need to know will be discussed. Section 3 discusses a common problem solving strategy, the importance of heuristics and metacognition in solving problems, and the importance of reflecting back after a problem is solved. Section 4 discusses the importance of supplemental problems and projects to teach problem solving.

2.0 Prerequisite Skills

Prerequisites for problem solving are those skills, knowledge and attitudes that are an important but often overlooked component of problem solving. Without these prerequisites, even students with general problem solving skills will have difficulty solving a problem.

2.1 Basic Knowledge

Research comparing the problem solving ability of novices and experts has shown the crucial role of subject-specific knowledge [7]. Without the crucial knowledge of a subject area, like physics or mathematics, a student will not be able to solve problems in that area.

To help students acquire the knowledge they need to solve problems, it is important to understand the different types of knowledge. Knowledge in an area is thought to consist of declarative, conceptual and procedural knowledge [8]. Declarative knowledge is comprised of facts, concepts and principles. Conceptual knowledge refers to the hierarchical network of knowledge and its corresponding relationships. Conceptual knowledge can be thought of as how a person has their knowledge organized in their mind. This knowledge reflects an understanding of principles that may be applied in a number of different contexts. Conceptual knowledge cannot be learned by rote, it must be obtained by thoughtful, reflective learning.

Procedural knowledge results from the organization of conceptual knowledge into rules or algorithms that can be used to solve specific problems (like solving a system of linear equations using Cramer's Rule). With practice, the rules become internalized and can be used automatically. Once the procedural knowledge has become automatic, it is rapidly accessed from memory and requires relatively little cognitive effort. After exposure to a large number and variety of problems, students begin to develop the knowledge needed to solve routinely a particular class of commonly occurring problems, sometimes referred to as a *schema* [8]. A knowledge base containing a carefully selected set of such problem schemata is very useful, since it allows one to decompose any problem into the readily solvable problems provided by these schemata.

To increase student's knowledge base, instructors should try to:

- Clearly relate new information to existing knowledge or other important concepts.
- Present the information in multiple formats (verbal, auditory, visual), to help students with different learning styles.
- Organize the information being presented hierarchically to approximate the way information is stored in memory.
- Use authentic (real-world) contexts for explanations, examples, and practice to help the learners relate what they learn to situations in which they will need to use the knowledge.
- Identify and teach the procedures needed to solve important basic kinds of problems.

2.2 Learning Skills

Learning skills refer to those skills students need to know in order to be successful in school. Yet, because they are so basic and they don't apply to any one class, these skills are rarely taught. Some skills that are important for students taking science and math classes are:

- Taking and organizing notes
- Reading a technical book
- Time management
- Writing assignments
- Study strategies

For more information on these skills, see the learning skills web site at the University of Victoria [9]. For each class, instructors should define several learning skills that they think are important for their class, talk about them at the beginning of the quarter, refer to these skills during the quarter and model the use of these skills when possible.

2.3 Motivation

The energy and persistence that a student shows in solving a problem is usually related to their motivation towards the course. Students that I have had in class that are really interested in learning the material, will spend hours on a project and talk about it outside of class. Yet too often students in mandatory courses are motivated to succeed in the class only because they need to pass the course as a part of their program's requirements. It is the fear of a bad grade that motivates them rather than the desire to learn something useful. The anxiety created by studying complex subject matter and the pressure to pass a course can seriously interfere with a student's memory and concentration. This will usually have a negative effect on a student's performance in class and it will leave them unable to think clearly when they try to solve problems.

For a good survey of methods to help motivate students see Davis [10].

2.4 Experience

Experience helps us to develop judgment as to the appropriateness of an answer and to estimate the size of quantities when the exact value is not known. This is different from the thinking skills used when learning new material. This is a memorization of *how big is big* and *how small is small*. Most experience factors come from the numerical data given in the various problems that we solve, the numerical answers we calculate, and any data that we have to look up [11].

One way to get students used to thinking about the size of quantities they have no experience with, is to introduce them to Fermi problems. A Fermi problem is simply a problem for which there is no apparent answer, yet with a few assumptions, a reasonable estimate can be made. An example of a Fermi problem is to estimate the number of piano tuners in New York City. A good source for Fermi problems is the University of Maryland Fermi Problems Site [12].

2.5 Communication and Group Skills

Education has traditionally emphasized individual performance, but the business world emphasizes teamwork and cooperation. Good teamwork skills do not come naturally for most people, yet many instructors seem to believe that forming groups of students and giving them something to do together, will be enough to enable them to function efficiently as a team.

To help students function better in a group, they should be made aware of the following skills:

- Listening – Listen to what other members of the team have to say. Use their ideas to develop new ideas. Don't try to dominate the discussion.
- Questioning – Ask questions when you don't understand what has been said. It is important for everyone to understand everything the group does at each step of the way.
- Persuading – In an exchange of ideas, it is important to defend your idea by carefully explaining it. Don't give in just because you think the other person is smarter, older, louder, etc.
- Respecting – It is important to respect and support the ideas and opinions of all members in your group.

- Helping – Help other members of your group who may have a hard time expressing themselves or do not catch on as quickly to a new idea.
- Sharing – It is important for every member of the team to share their ideas.
- Participating – Everyone needs to contribute to the work that is required to get a project finished.

For more information on developing group skills, see the Group Skills Development Pledge at Indiana University [13].

In forming groups for classroom work, researchers investigating group problem solving have found that [14, 15]:

- The optimal group size is three students. With two people, there is often not enough knowledge to solve the problem, and with four people, one member tends to be left out of the process.
- The instructor should assign members so that each group has a mixture of abilities. That way, the stronger students can reinforce their knowledge by explaining concepts to the weaker students and weaker students can see how better students go about solving problems. It has also been found that groups of two men and one woman did not work well, particularly at the beginning of the course. The men tend to ignore the woman, even if she is the highest ability student in the group. Until instructors get to know their students well, it is safest to assign groups of three men, three women, or two women and one man.
- Groups should not be changed too often because students need some time to get comfortable working with each other. Yet groups should be changed often enough so students realize they can make any group successful. To ease student resentment to changing groups, it is useful to tell them that:
    - You want them to get to know other people in the class, so we will change groups often. By the end of the quarter, they should have worked with many people in the class. This helps build a sense of community.
    - No matter what career they enter, they will have to work cooperatively with many different kinds of people, not just their friends. So they need to learn how to work successfully in groups.
- Assign team roles that rotate with each assignment. The roles correspond to the planning and monitoring strategies individuals must perform independently when solving problems.
    - Manager – organizes the assignment into subtasks, allocates responsibilities, and keeps the group on task. The manager must ensure that everyone in the group participates and contributes.
    - Checker/Recorder – takes notes as work progresses and writes the final report or problem solution. The checker/recorder must ensure that all group members can explicitly explain how the problem was solved.
    - Skeptic – questions premises and plans. The skeptic plays the role of devil's advocate, suggests alternative possibilities and keeps the group from leaping to premature conclusions.
- Teams should periodically assess how well they are working together.
- Make sure seats are arranged so students are facing each other.

3.0 Problem Solving Skills

While the skills in the previous section are basic skills suitable for any class, the skills in this section are specific to problem solving.

3.1 Strategy

Even if students have all of the basic knowledge and background skills they need to solve a problem that still does not guarantee success when trying to solve a problem they have never seen before. An additional element that is needed is for students to have a general strategy that they can use for problem solving. A *strategy* is a set of sequential steps (or procedure) used by a problem solver in arriving at a solution. The strategy should help students by guiding them to efficiently extract relevant data from the question and by giving them a planned approach to solving the problem. One common problem solving strategy is composed of the following general steps [16]:

- Define the Problem. Identify the actual problem you are trying to solve.
- Think about the Problem. What are the attributes of the problem? Identify the area of knowledge involved. Collect information.
- Plan a Solution. Think of alternate ways to solve the problem. Flowchart a solution.
- Carry out the Plan. Solve the problem.
- Look Back. Verify that the problem solved was the one originally defined. Check reasonableness and math. Check criteria and constraints. Communicate results.

3.2 Heuristics

Heuristics are general suggestions or “rules of thumb” that are useful in solving a great variety of problems. Heuristics are powerful and general, but they are not guaranteed to work. That is why there are so many of them – if one does not work, you need to try another.

General heuristics are usually context free and apply across many different situations. Probably the most common general heuristic is the ‘means-end analysis’. Simply stated, this heuristic says to do something to get a little closer to your goal. For other general heuristics see [17, 18]. Specific heuristics are used in specialized areas, like applying the conservation of momentum principle to solve collision problems in physics, or telling students to *check the units*, *neglect small terms*, or *use crude approximations*.

A curriculum that encourages problem solving needs to provide more than just practice in solving problems; it needs to offer explicit instruction in the nature and use of heuristics.

3.3 Metacognition

All heuristics help break down a problem into pieces. If a large problem is broken down into pieces, then one challenge becomes keeping track of what to do and when. The importance of monitoring sub-goals is an example of a more general phenomenon: the capacity to examine and control one's own thoughts. This self-monitoring is known as metacognition [19]. Metacognition is essential for any extended activity, especially problem solving, because the problem solver needs to be aware of the current activity, of the overall goal, the strategies used to attain the goal and the effectiveness of those strategies.

Teachers cannot simply assume that students will engage in metacognition, it must be taught explicitly as an integral component of problem solving. To help students understand metacognition and to become aware of their own use of metacognition, instructors can

- discuss the importance of metacognitive knowledge and regulation. Ideally, such a discussion helps students construct an explicit mental model of the self-regulation process.
- model their own metacognition as they are solving a problem. Too often teachers discuss and model their cognition (i.e., how to perform a task) without modeling metacognition (i.e., how they think about and monitor their performance).
- provide time for group discussion and reflection.
- introduce Thinking Aloud Paired Problem Solving [20]. This is a process where two students work together to solve a series of problems. One student becomes the problem solver and they say everything they are thinking about as they work through a problem. The other student is the listener, whose objective is to understand every step the problem solver is making and to ask questions if they don't understand something, if they see the problem solver has made a mistake, or if the problem solver gets stuck.
- tell students to keep in mind three questions as they are working on a problem [21]:
    - What (exactly) am I doing? (Can I describe it precisely?)
    - Why am I doing it? (How does it fit into the solution?)
    - How does it help me? (What will I do with the outcome when I obtain it?)

3.4 Ability to create, to generalize and to simplify

Once a problem is solved, students miss out on an important learning opportunity if they just hand in their work and then forget about it. The period after a problem has been solved has been identified by Polya [22] as a key moment in time when significant learning can take place. It is what you do after you have solved a problem that really determines how much you learn from a problem. The type of activities that students should be encouraged to undertake to promote learning from problem solving are [23, 24]:

- Extending processes - Review the process that was used to solve the problem. How could the problem have been solved another way? If the problem was solved analytically, how could it have been done numerically?

- Extending solutions - What will happen to the solution if an important parameter is made a little bigger or a little smaller? What will a graph look like showing the result as a function of an important parameter?
- Extending problems – To solve the problem certain assumptions were made. What will change in the problem if different assumptions are made?
- Stating a new problem – Can you think of another problem that could be solved in a similar way? For example, in physics mass-spring-damper problems are described by the same equations as RLC circuit problems.
- Self-reflection – What did you learn from solving this problem? Record the experience factors you used or calculated in the problem.

4.0 Supplemental Problems and Projects

Textbook problems are good for giving students practice in learning new material, but they are not useful for teaching problem solving. In order to be proficient in problem solving, student need to be given problems that they don't have any idea of how they are going to solve when they first see the problem. Most textbook problems don't fall into this category because:

- Students know the problem is related to the section they just read. So they reread the section (or read it for the first time), searching for a problem similar to the one they are trying to solve.
- The statement of the problem provides everything needed to solve the problem with no missing or extraneous information to distract them.
- Textbook problems are often unrealistic.
- Problems typically rely on just one concept – they are not long multi-part problems.
- Textbook problems tell students what to solve for, i.e. find the amount of money in the savings account after five years, or what was the initial speed of the car? Yet finding the proper quantity to solve for is often a real challenge in an actual problem.
- Assumptions are clearly spelled out, robbing students of the need to make a decision.

To overcome the problems with textbook assignments, instructors need to give non-textbook based assignments to their students. Depending on the type of problems and projects assigned, the benefits to students of working on these supplemental problems and projects is that students will:

- have an improved ability to analyze a problem and determine what are the key issues and what background information is needed to solve the problem;
- be better able to take a problem and formulate a mathematical model of it;
- be more adept at selecting the appropriate mathematics to solve a problem;
- be able to successfully interpret the solution in terms of the original problem;
- be better at writing a technical paper;
- be better at orally communicating the results of a scientific study;
- be more engaged by the material and find an increased interest in the subject; and

- be able to see how the mathematics they are studying can be used to solve real problems.

There are many different kinds of problems and projects that can be used, depending on an instructor's educational objectives. Some of the different types of projects that can be assigned are:

- Theoretical mathematics projects – Based on reading a number of resources, write a paper on a mathematical topic talked about briefly or not at all in class. (i.e., compare different methods for solving a system of linear equations.)
- Context Rich Problems – Context rich problems are word problems that are more difficult than typical story problems. Characteristics that make these problems better for practicing problem solving are [25]:
    - The problems are structured so that it is necessary to make decisions on how to proceed with the problem.
    - All the required information may not be given, for example the weight of a car or a distance that may be easily estimated. Also, extra information may be given that is not required to solve the problem.
    - The problem should be relevant to the lives of the students.
    - The problem should not depend on students knowing a trick nor should they be mathematically tedious.

  To help in creating new problems, the University of Minnesota Physics Education Group has developed some guidelines on how to make a problem more difficult [26], how to judge the complexity of a problem [27] and how to write a good group problem [28].
- Experimental Problems – These problems require students to compare their theoretical prediction of how a system behaves with experimental data that they collect.
- Interdisciplinary Lively Application Projects (ILAP's) – ILAP's are small group projects developed by faculty in mathematics, science and engineering. ILAP's require students to use scientific and quantitative reasoning, mathematical modeling, symbolic manipulation, and computation to explore, solve, and analyze scenarios, issues, and questions [29].
- Application Projects – Find a problem and create a mathematical model that will address the problem. (i.e., investigate how projectile motion is affected by air resistance.)
- Article Review – Find a journal article that describes a mathematical model in a field of interest. Prepare a report that describes the problem and the mathematical model at a level that will be understandable to your classmates. (i.e., AIDS model).

Different sources of supplemental problems and projects are:

1. To create context rich problems by adapting textbook problems. Examples of context rich problems in physics can be seen at:
    - Context Rich Problems On Line Archive from the University of Minnesota, http://groups.physics.umn.edu/physed/Research/CRP/on-lineArchive/ola.html

2. Journals:
   - The American Mathematical Monthly, http://www.maa.org/pubs/monthly.html
   - The College Math Journal, http://www.maa.org/pubs/cmj_march04.html
   - UMAP Journal, http://www.comap.com/undergraduate/products/
   - Journal of Online Mathematics and its Applications: http://www.joma.org/
   - Teaching Mathematics and its Applications, http://www3.oup.co.uk/teamat/scope/
   - PRIMUS (Problems, Resource and Issues in Undergraduate Mathematics Studies), http://www.dean.usma.edu/math/pubs/primus/default.htm

3. Problems from past mathematical modeling contests:
   - The Mathematical Contest in Modeling, http://www.comap.com/undergraduate/contests/mcm/previous-contests.php
   - The High School Contest in Mathematical Modeling, http://www.comap.com/highschool/contests/himcm/previous%20problems.html

4. Projects developed by different organizations:
   - IDEA is **I**nternet **D**ifferential **E**quations **A**ctivities, an interdisciplinary effort to provide students and teachers around the world with computer based activities for differential equations in a wide variety of disciplines. The web site currently lists 18 differential equations projects. http://www.sci.wsu.edu/idea/welcome.html
   - Mathematical Modeling in a Real and Complex World, The Connected Curriculum Project. A series of projects covering discrete, continuous and probabilistic models. http://www.math.montana.edu/frankw/ccp/modeling/topic.htm
   - Stepping Stones to Mathematical Modeling. Forty modeling projects are listed that have been written and tested by high school teachers under the direction of university mathematics and education professors. http://www.indiana.edu/~hmathmod/
   - Project Intermath. Sixty-two modeling projects (ILAP's) are listed that have been created by professors from science, engineering, mathematics and computer science departments from around the country. The goal of the projects are to demonstrate the interdependence of mathematics and science. http://www.projectintermath.org/products/listing/
   - Reform Calculus Resources lists numerous activities for a calculus class along with seven long term projects. http://barzilai.org/archive/
   - University of Minnesota Calculus Initiative lists four modules that emphasize geometric concepts of calculus while examining applications of mathematics to the physical and life sciences. http://www.geom.uiuc.edu/education/calc-init/
   - Computational Science in Education. Fifty-one modeling projects covering a variety of science areas. http://www.ncsec.org/models.cfm

- Classroom Projects with Environmental Applications, for pre-calculus and calculus classes. http://earthmath.kennesaw.edu/main_site/index.htm
- Mathematical Models with Applications, Science and Engineering Module, http://www.tenet.edu/teks/mmacd/curric/units2.htm
- Project Links, Mathematics and its Applications in Engineering and Science, http://links.math.rpi.edu

5. Have students find their own set of data and construct a mathematical model to describe the data:
   - StatLib Data Sets archive http://lib.stat.cmu.edu/datasets/
   - Statistical Science Web Data Sets http://www.statsci.org/datasets.html
   - Whatcom Community College, look under the heading of *Real Data*, http://math.whatcom.ctc.edu/

6. Look at web sites of instructors teaching a similar class:
   - Galvin's Calculus Projects, from classes taught by Gavin LaRose at the University of Michigan. http://www.math.lsa.umich.edu/~glarose/courseinfo/calc/calcprojects.html
   - Student Projects in Multivariable Calculus, from classes taught by David Arnold at College of the Redwoods. http://online.redwoods.cc.ca.us/instruct/darnold/CalcProj/Index.htm
   - The Freedonian Farnsworth's Sparrow ecosystem project for a differential equations class by Mark Janeba http://www.willamette.edu/~mjaneba/courses/ma142/s97/project2.html
   - Tommy Ratliff of Wheaton College has projects listed for his calculus classes, http://www2.wheatonma.edu/academic/academicdept/MathCS/Faculty/tratliff/writing/home.html
   - Mathematical Modules in Biology and Chemistry by Meghan A. Burke and Sean F. Ellermeyer, http://science.kennesaw.edu/~mburke/modules/

7. Experimental Problems:
   - *Physics Teacher*, Published by the American Association of Physics Teachers.
   - Experiment Problems in Mechanics, Electricity and Magnetism, by Alan Van Heuvlan, http://www.physics.ohio-state.edu/~physedu/index2.html

8. Books on mathematical modeling:
   - *A First Course in Mathematical Modeling*, Third Edition, Frank Giordano, Maurice Weir, William Fox, Brooks/Cole, 2003.
   - *Environmental Mathematics in the Classroom* by B.Fusaro, Cambridge University Press, 2003.
   - *Guide to Mathematical Modeling*, Dilwyn Edwards and Mike Hamson, CRC Mathematical Guides, CRC Press, 1990.
   - *Mathematics in Action: Modelling the Real World*, Richard Beare. Charwell-Bratt Publisher.

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- *Mathematical Modelling*, J. Berry and K. Houston, Edward Arnold Publishing, 1995.
- *Mathematical Modeling*, Mark Meerschaert, Academic Press, 1993.
- *Mathematical Modeling in the Environment*, Charles Hadlock, The Mathematical Association of America, 1998.
- *A Course in Mathematical Modeling*, Douglas Mooney and Randall Swift, The Mathematical Association of America, 1999.
- *Mathematical Models in the Social Sciences*, John Kemeny and J. Laurie Snell, The MIT Press, 1978.

9. Books describing mathematics projects:
    - *Calculus Mysteries and Thrillers*, R.Grant Woods, MAA, 1998
    - *Interdisciplinary Lively Application Projects (ILAPs)*, David G. Arney, MAA, 1997
    - *Applications of Calculus*, Philip Straffin, Ed., MAA Notes 29, 1993.
    - *Calculus Problems for a New Century*, Robert Fraga, Ed., MAA Notes 28, 1993.
    - *Calculus: The Dynamics of Change*, Roberts, A. Wayne, Ed., MAA Notes 39, 1996
    - *Resources for Teaching Linear Algebra*, David Carlson, Charles Johnson, David Lay, Duane Porter, Ann Watkins, William Watkins Ed., MAA Notes 42, 1997.
    - *Revolutions in Differential Equations*, Michael J. Kallaher, Ed., MAA Notes 50, 1999.

10. Applications at developers of software packages:
    - Maple Application Center, http://www.maplesoft.com/applications/index.aspx
    - Mathematica Infocenter, http://library.wolfram.com/infocenter/
    - Matlab Central, http://www.mathworks.com/matlabcentral/link_exchange/

11. For students interested in mathematical research, the following resources are useful:
    - Martin Gardner's books
    - Ian Stewart's books and column in Scientific American Magazine
    - *Mathematics Teacher* published by the National Council of Teachers of Mathematics
    - *College Mathematics* and *Mathematics Magazine* published by the American Mathematical Association
    - *Journal of Recreational Mathematics*, Published by Baywood Publishing Company, 26 Austin Avenue, Amityville, New York 11701
    - *Student Research Projects in Calculus*, Cohen, Gaughan, Knoebel, Kurtz, American Mathematical Association, 1991.

12. Publications from conferences on mathematical modeling in the undergraduate curriculum:

- *Teaching and Applying Mathematical Modelling*, Berry JS, Burghes DN, Huntley ID, James DJG and Moscardini AO, editors, 1984, Ellis Horwood, Chichester.
- *Mathematical Modelling Methodology, Models and Micros*, Berry JS, Burghes DN, Huntley ID, James DJG and Moscardini AO, editors, 1986, Ellis Horwood, Chichester.
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- *Applications and Modelling in Learning and Teaching Mathematics*, Blum W, Berry JS, Biehler R, Huntley ID, Kaiser-Messmer G and Profke L, editors, 1989, Ellis Horwood, Chichester.
- *Teaching of Mathematical Modelling and Applications*, Niss M, Blum W and Huntley ID, editors, 1991, Ellis Horwood, Chichester.
- *Innovation in Maths Education by Modelling and Applications*, De Lange J, Keitel C, Huntley ID and Niss M, editors, 1993, Ellis Horwood, Chichester.
- *Advances and Perspectives in the Teaching of Mathematical Modelling and Applications*, Sloyer C, Blum W and Huntley ID, 1995, editors, Water Street Mathematics, Yorklyn, Delaware.
- *Teaching and Learning Mathematical Modelling*, Houston SK, Blum W, Huntley ID and Neill NT, editors, 1997, Albion Publishing Ltd (now Horwood Publishing Ltd.), Chichester.
- *Mathematical Modelling, Teaching and Assessment in a Technology-Rich World*, Galbraith P, Blum W, Booker G and Huntley ID, editors, 1998, Horwood Publishing Ltd., Chichester.
- *Modelling and Mathematics Education: ICTMA 9: - Applications in Science and Technology*, Matos JF, Blum W, Houston SK and Carreiara SP, editors, 2001, Horwood Publishing, Chichester.
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- *Proceedings of the 1996 Conference on Mathematical Modeling in the Undergraduate Curriculum*, Mathematics Department, University of Wisconsin-La Crosse, La Crosse, WI 54601. http://perth.uwlax.edu/mathematics/conference/proceedings.html

5.0 Conclusion

"In trying to implement new ways to teach problem solving, resistance to change usually comes in one of two ways. Either professors and students believe that present educational practices are producing satisfactory outcomes – thus, there is no compelling need for change and efforts to promote change prompt opposition. Or there is no time to cover anything new - teachers complain *I need to cover content* and students say *just tell me how to get the right answer*"[30]. Studies have shown that the current educational system is not teaching problem solving [3,4]. So, if we truly believe that problem solving is a worthwhile educational goal for our students, we have to find the time to properly teach it. If we don't, we are doing a disservice to our students and our community.

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